# SIR Models

## A Brief Discussion

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# SIR Models

- SIR models are a type of compartment model that can be used to model epidemics.
- The total population of some area is divided into
  - Susceptible individuals who can be infected $S = S(t)$
  - Infected individuals $I = I(t)$
  - Recovered individuals who are immune to infection $R = R(t)$
- For the moment we ignore birth as well as death from other causes, but infected individuals can die from the disease.

# Transitions between Compartments

- Transitions between $S$ and $I$ in the simplest model are proportional to the product of $SI/N$, where $N = S + I + R$; this assumes a completely mixing population which is an enormous simplification but can be useful.
- Imagine that each infected person makes contact with $k$ others and has a probabiity $p$ of passing on enough virus for infection. The expected number of contacts with susceptible individuals is $k(S/N)$ so the total number of new infections is $\beta IS/N$, where $\beta = pk$.

# Transitions between Compartments

- We assume that a fraction $\gamma$ of infected individuals transitions to recovered $R$ each period and a fraction $\delta$ dies.
- For extra simplicity, we can assume the period is the length of infection so that $\gamma + \delta = 1$.

$$
\begin{aligned}
S_{t+1} &= S_t - \beta I_t S_t / N_t \\
I_{t+1} &= I_t + \beta I_t S_t / N_t - (\gamma + \delta) I_t \\
R_{t+1} &= R_t + \gamma I_t \\
N_{t+1} &= N_t - \delta I_t
\end{aligned}
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The number of new infections per initial infection $\beta = kp$ must be greater than 1 else there is no epidemic. Infections increase if $\beta S/N > 1$. Herd immunity is the process if constant reduction in $\beta S/N$. Infections stop increasing if this is 1 or less, though may persist for a long time.

# Complications

- The "stable" situation is when $\beta S/N \approx 1$, but dynamics overshoots.
- Populations are complex networks of interaction, not random mixing.
- The number of contacts $k$ varies dramatically across individuals and can be substantially affected by social distancing policies by governments or populations.
- Vaccination adds an additional process that moves people from $S$ to $R$ without infection.

# Complications

The infection fatality rate, IFR $\delta$ for COVID will vary substantially by age and co-morbidities. Here are some estimates.

| Age | Risk % | Per 100,000 |
|---|---|---|
| 10 | 0.002 | 2 |
| 25 | 0.01 | 10 |
| 55 | 0.4 | 400 |
| 65 | 1.4 | 1,400 |
| 75 | 4.6 | 4,600 |
| 85 | 15 | 15,000 |

# Complications

- The case fatality rate CFR is always higher than the IFR
- The ratio between the two is also not constant, with a substantial dependence on age.
- This is about 15:1 in the 30–49 age group, 7:1 in the 50–69 age group and 5:1 in the 70–79 age group.
- So a 1.4% IFR at age 65 is about a 10% CFR and the 4.6% IFR at age 75 is about a 23% CFR.

- These models can be used for epidemic forecasting and analysis, but there are numerous facts that need to be know to make this work.
- Almost none of these facts was known a year ago, and even now knowledge is incomplete.
- And SARS2 variants can have different infectivity profiles and possibly different mortality rates.
- This has been very difficult in the last year, and overconfidence has frequently been the cause of substantial error.